We present a method for and results from the determination of the limit heattransfer power of a thermal tube with a segmented artery.

Arterial thermal tubes (TT) make it possible, in a number of cases, to obtain higher characteristics than in the case of TT with capillary systems (CS) of other types; however, the methods of calculating these extreme flows of heat have not been adequately developed. No consideration has been given in published papers to the effect of the force of gravity on $\mathrm{Q}_{\text {max }}$ TT, nor of the excess of the heat-carrying coolant, the existence of orifices within the arterial wall for the egress of vapor, and for the unique features involved in the structure of the CS. These factors are taken into consideration in the present paper. We examine a TT with a segmented artery [1], formed by means of a longitudinal trough-shaped insert welded to the housing wall (Fig. 1).

The flow of heat with which the drying out of the threading groove at the end of the vaporization zone begins is regarded as the limit heat flow. It was assumed that $Q_{\max } T T$ is limited by the available capillary pressure pulse, that the flow of the heat-carrying coolant is laminar, that the interaction between the liquid and the vapor is insignificant, that the flow of heat is fed in and removed from segments corresponding to the angle $2\left(\pi-\varphi_{c}\right)$, and that the heat-flux density is constant. In analogy with the manner in which this was done in [2], the coolant circulation contour was divided into two parts: 1) the threaded grooves in the zone of heat supply, including a segment with a flanged arterial wall; 2) the remaining elements of the CS and the vapor channel. For each of these parts we have calculated $Q_{\max i}$ for various values of the meniscus radius in the CS at the junction between the parts of the coolant circulation contour. We have constructed graphs (see Fig. 2) on the basis of these calculation results. If at the point of intersection between curves 1 and 2 we have $\mathrm{R}_{M^{\prime}} \geq \mathrm{d}_{\mathrm{ori}} / 4 \cos \theta$, then $\mathrm{Q}_{\max } \mathrm{TT}$ corresponds to the point of intersection (Fig. 2a). Since it is impossible to achieve a meniscus with a radius smaller than $d_{o r i} / 4 \cos \theta$, when $R_{M}$ < $\mathrm{d}_{\text {ori }} / 4 \cos \theta, \mathrm{Q}_{\max T \mathrm{~T}}$ is found in the manner shown in Fig. 2b. To calculate $\mathrm{Q}_{\max 1}$ and $\mathrm{Q}_{\max 2}$ we solved the pressure-balance equations. The pressure losses contained in these equations for those segments of the coolant circulating contour were determined on the basis of considerations presented below.

For the threading grooves in the heat inflow and outflow zones

$$
\begin{equation*}
\frac{d P}{d \varphi}=\frac{1}{2} \frac{v_{1} \Pi_{\mathrm{t} \varphi}^{2}}{F_{\mathrm{t} \varphi}^{3}} d_{\mathrm{in}} \frac{Q S_{\mathrm{p}}}{l_{\text {in(out })}^{L}} \frac{\pi-\varphi}{\pi-\varphi_{\cdot \mathrm{B}}} \pm\left(\rho_{1}-\rho_{\mathrm{v}}\right) \frac{d_{\mathrm{in}}}{2} g \frac{d}{d \varphi}(\cos \varphi) . \tag{1}
\end{equation*}
$$

For the zone of TT heat outflow a minus sign must appear in front of the second term in the right-hand side of (1), while a plus sign must appear for the heat-supply zone. The area of and the wetted perimeter of the liquid in the lateral cross section of the threaded groove, corresponding to the angle $\varphi$, are determined from the formulas

$$
\begin{gather*}
F_{\mathrm{t} \varphi}=\frac{r^{2}}{2}(\pi-2 \alpha+\sin 2 \alpha)+2 r\left(H_{\mathrm{m}}-r\right) \cos \alpha+\left[\left(H_{\mathrm{m}}-r\right)+r \sin \alpha\right]^{2} \operatorname{tg} \alpha-R_{\mathrm{m}}^{2}[\beta-\sin \beta \cos \beta]  \tag{2}\\
\cdot \quad \Pi_{\mathrm{t} \varphi}=2\left[\frac{H_{\mathrm{m}}-r(1-\sin \alpha)}{\cos \alpha}+r\left(\frac{\pi}{2}-\alpha\right)\right] \tag{3}
\end{gather*}
$$

When $R_{\text {max }} \geq R_{m} \geq R_{g}, H_{m}=H_{t}$

$$
\begin{equation*}
\beta=\arcsin \left[\frac{\left(H_{\mathrm{t}}+r / \sin \alpha-r\right)}{R_{\mathrm{m}}}\right] \tag{4}
\end{equation*}
$$

[^0]

Fig. 1. Lateral TT cross section: 1) TT casing; 2) longitudinal insert, artery wall.



Fig. 2. Limit flows of heat from two parts (1, 2, respectively) of the heat-carrying circulation contour as functions of the meniscus radius at the point of contact between these two parts.


Fig. 3. Longitudinal cross section of the TT casing in the "puddling" zone.

If $R_{\text {min }} \leq R_{m}<R_{g}$,

$$
\begin{equation*}
\beta=\left[\frac{\pi}{2}-(\alpha+\theta)\right],{ }_{\mathrm{m}} H=\frac{R_{\mathrm{m}} \sin \beta}{\operatorname{tg} \alpha}-\frac{r}{\sin \alpha}+r . \tag{5}
\end{equation*}
$$

The minimum radius of the meniscus at the inlet to the groove profile is given by

$$
\begin{equation*}
R_{\mathrm{g}}=\frac{\left(H_{\mathrm{t}}+r / \sin \alpha-r\right) \operatorname{tg} \alpha}{\cos (\alpha+\theta)} \tag{6}
\end{equation*}
$$

the minimum radius of the liquid in the groove is

$$
\begin{equation*}
R_{\mathrm{mtn}}=\frac{r \cos \alpha}{\cos (\alpha+\theta)} \tag{7}
\end{equation*}
$$

and the maximum radius of curvature for the surface of the liquid in the thermal tube is

$$
R_{\max }=\infty \text { when } g=9,81 \mathrm{~m} / \mathrm{sec}^{2},
$$

$$
\begin{equation*}
R_{\max }=\frac{d_{\mathrm{in}}\left(d_{\mathrm{in}}-H_{\mathrm{ar}}-\delta_{\mathrm{w}}\right)}{2 d_{\mathrm{in}}-H_{\mathrm{ar}}-\delta_{\mathrm{w}}} \text { when } g=0 . \tag{8}
\end{equation*}
$$

For those segments of the threading groove beneath the flange of the arterial wall (between points A and B )

$$
\begin{equation*}
\Delta P_{\mathrm{f} 1 \mathrm{a}}=\frac{v_{1} \cdot d_{\mathrm{in}}\left(\varphi_{B}-\varphi_{A}\right) Q_{\mathrm{max}_{i}}}{2 L l_{\mathrm{in} \boldsymbol{n}}\left[F_{\mathrm{t} \max }^{3}\left(\left(S_{\mathrm{t}} \Pi_{\mathrm{i}}^{2}\right)+\delta_{\mathrm{w}} \varepsilon_{\mathrm{w}} \cdot d_{\mathrm{h} . \mathrm{w}}^{2} / 16\right]\right.}, \tag{9}
\end{equation*}
$$

$\ell_{i n}$ is the length of the corresponding zone (for the supply or removal of heat);

$$
\begin{gather*}
F_{\mathrm{t} \max }^{\prime}=\left(H_{\mathrm{t}}+\frac{r}{\sin \alpha}-r\right)^{2} \operatorname{tg} \alpha-r^{2}\left(\operatorname{ctg} \alpha-\frac{\pi}{2}+\alpha\right),  \tag{10}\\
\Pi_{1}=2\left[\frac{H_{\mathrm{t}}(1+\sin \alpha)}{\cos \alpha}+r\left(\frac{\pi}{2}-\alpha\right)\right] . \tag{11}
\end{gather*}
$$

For the artery and the vapor channel

$$
\begin{equation*}
\Delta P_{\mathrm{ar}}=\frac{2 v_{1} \Pi_{\mathrm{ar}}^{2}}{F_{\mathrm{ar}}^{3}} \frac{Q_{\max 1}}{L} l_{\mathrm{ar}} ; \quad \Delta P_{\mathrm{v}}=\frac{2 v_{\mathrm{v}} \Pi_{\mathrm{v}}^{2}}{F_{\mathrm{v}}^{3}} l_{\mathrm{ar}}, \tag{12}
\end{equation*}
$$

where the effective length of the artery

$$
\begin{equation*}
l_{\mathrm{ar}}=l_{\mathrm{TT}}-0,5\left(l_{\mathrm{in}}+l_{\text {out }}\right. \tag{13}
\end{equation*}
$$

Under the conditions in which the force of gravity is effective the excess of the liquid forms a "puddle" within the cavity of the tube. The flow within the "puddle" comes about as a consequence of the gravitational thrust. Therefore, in calculating $\mathbb{Q}_{\text {max }}$, resulting from the capillary limitation, we can exclude that segment occupied by the "puddle" from the circuit containing the coolant circulation tract. The dashed line in Fig. 3 represents the outline of the "puddle" surface under isothermal conditions, while the solid line identifies the operational TT. The length $\mathrm{x}_{0}$ of the "puddle" can be determined through solution of Eqs. (14)-(16) with the following boundary conditions: $x=0, \varphi=\varphi_{a} ; x=x_{0}, V=V_{\text {exc }}$ :

$$
\begin{gather*}
\frac{d y}{d x}=\frac{d_{\mathrm{in}}}{2} \sin \varphi_{1} \frac{d \varphi_{1}}{d x},  \tag{14}\\
\frac{d P}{d x}=\frac{2 v_{1} Q_{\mathrm{max}_{i} k_{x} \Pi_{x}^{2}}^{F_{x}^{3} L}=\left(\rho_{1}-\rho_{\mathrm{I}}\right) g\left(\frac{d y}{d x} \cos \psi-\sin \psi\right),}{}  \tag{15}\\
\frac{d V}{d x}=F_{x}-F_{\mathrm{ar}}, \tag{16}
\end{gather*}
$$

where $k_{x}=1$ when $x<\left(x_{0}-\ell_{f 1 a}\right)$; $k_{x}=\left(x_{0}-x\right) / \ell_{f l a}$ when $x>\left(x_{0}-\ell_{f 1 a}\right)$.
The area and wetted perimeter of the transverse cross section of the liquid in the "puddle" zone

$$
\begin{gather*}
F_{x}=\frac{d_{\mathrm{in}}^{2}}{8}\left(2 \varphi_{\mathrm{p}}-\sin 2 \varphi_{\mathrm{p}}\right),  \tag{17}\\
\Pi_{x}=d_{\mathrm{in}}\left(\varphi_{\mathrm{p}}+2 \sin \varphi_{\mathrm{p}}\right) . \tag{18}
\end{gather*}
$$

The area of the transverse cross section of the artery

$$
\begin{equation*}
F_{\mathrm{ar}}=\frac{d_{\mathrm{in}}^{2}}{8}\left(2 \varphi_{\mathrm{a}}-\sin 2 \varphi_{\mathrm{a}}\right), \tag{19}
\end{equation*}
$$

the volume of the excess heat carrier

$$
\begin{equation*}
V_{\mathrm{exc}}=\left(m_{\mathrm{pr}}-V_{\mathrm{v}} \rho_{\mathrm{v}}-V_{\mathrm{CS}} \rho_{\mathrm{I}}\right) \frac{1}{\rho_{1}} . \tag{20}
\end{equation*}
$$

The losses that arise in the flow of the heat carrier are offset by the pressure pulses in the capillaries and due to gravity. These pulses in individual sections of the circulation tract for the heat carrier are determined from the relationship


Fig. 4. Limit $T T$ heat flow as functions of temperature: 1 , 2) $\mathrm{g}=0 ; 3$, 4) $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{sec}^{2}$. $\mathrm{Q}_{\max } \mathrm{TT}, \mathrm{W} ; \mathrm{T}, \mathrm{K}$.

$$
\begin{align*}
& \text { 1) } \Delta P_{\mathrm{t} 1}=\sigma\left(\frac{1}{R_{\mathrm{m} \cdot \mathrm{j} \cdot \mathrm{i}}}-\frac{2}{R_{\text {max }}}\right)-g\left(\rho_{1}-\rho_{\mathrm{v}}\right)\left(l_{\mathrm{TT}}-x_{0}\right) \sin \psi  \tag{21}\\
& \text { 2) } \Delta P_{\mathrm{t} 2}=\sigma\left(\frac{1}{R_{\min }}-\frac{1}{R_{\mathrm{m} \cdot \mathrm{j}_{i}}}\right) \tag{22}
\end{align*}
$$

Different methods are applicable to the calculation under conditions $\mathrm{g}=0$ and $\mathrm{g}=9.81 \mathrm{~m} /$ $\sec ^{2}$.

When $g=0$, in order to find the relationship between the limit flow of heat in the initial portion of the heat-carrier circulation tract and the magnitude of the meniscus radius at the joint between the first and second parts of the circulation contour, i.e., $Q_{\max }=$ $f\left(R_{m . j_{i}}\right)$, for each of the several values of $R_{m .} j_{i}$ a series of values for $Q_{i}$ are specified and for each of these on the basis of formulas (9)-(13) we determine the pressure differences across the artery, the segments of the CS, in the arterial wall flange zones, as well as in the vapor channel, and their total:

$$
\begin{equation*}
\Delta P_{1_{i}}=\Delta P_{\mathrm{av}_{i}}+\Delta P_{\mathrm{v}_{i}}+\Delta P_{\text {flan }_{i}} . \tag{23}
\end{equation*}
$$

We then determine the radius of the meniscus in the threading groove situated at the end of the heat-removal zone at the edge of the arterial wall flange

$$
\begin{equation*}
R_{\mathrm{m}_{i}}=\left(\frac{\Delta P_{1_{i}}}{\sigma}-\frac{2}{d_{\mathrm{in}}}\right)^{-1} . \tag{24}
\end{equation*}
$$

Having solved Eq. (1) with boundary conditions $\varphi=\pi$

$$
\begin{equation*}
R_{\mathrm{in}}=\left(\frac{2 \cos \theta}{R_{\max }}-\frac{2}{d_{\mathrm{in}}}\right)^{-1} ; \quad \varphi=\varphi_{\mathrm{B}^{\prime}} \quad R_{\mathrm{m}}=R_{\mathrm{m}_{i}}, \tag{25}
\end{equation*}
$$

we determine the pressure difference across the threading grooves of the heat-removal zone, i.e., $\Delta \mathrm{P}_{\mathrm{t}} \mathrm{fl}_{\mathrm{i}}, \mathrm{Q}_{\max 1}$, and namely, that value of $\mathrm{Q}_{\mathrm{i}}$ at which the following condition is satisfied:

$$
\begin{equation*}
\Delta P_{1_{i}}+\Delta P_{\mathrm{fla}}=\Delta P_{\mathrm{t} 1} \tag{26}
\end{equation*}
$$

In order to find the relationship $Q_{\max 2}=f\left(R_{m_{i}}\right)$, we solve Eqs. (1) and (22) jointly, with the boundary conditions

$$
\begin{equation*}
\varphi=\pi, \quad R_{\mathrm{in}}=R_{\mathrm{m} 1 \mathrm{n}}, \quad \varphi=\varphi_{\mathrm{B}}, \quad R_{\mathrm{m}}=\left(\frac{\Delta P_{\mathrm{f} 1 \mathrm{a} 2}}{\sigma}+\frac{1}{R_{\mathrm{m} \cdot \mathrm{j} \cdot \mathrm{i}} .}\right)^{-1}, \tag{27}
\end{equation*}
$$

where $\Delta \mathrm{P}_{\mathrm{fla}}^{\mathrm{i}} \mathrm{i}$ denotes the loss of pressure in the CS segment which includes the arterial wall flange in the heat-inflow zone.

Based on the found relationships $Q_{\max 1}=f\left(R_{m} \cdot j_{i}\right)$ and $Q_{\max 2}=f\left(R_{m} . j_{i}\right)$ we construct graphs (see Fig. 2) and find $\mathrm{Q}_{\text {max }}$ TT.

When $g=9.81 \mathrm{~m} / \mathrm{sec}^{2}$, we specify a number of values for the length $\mathrm{x}_{0 \mathrm{i}}$ of the puddle and for each of these we calculated $\mathrm{Q}_{\max { }_{1}}$ on the basis of the formula

$$
\begin{equation*}
Q_{\max 1_{i}}=\frac{\Delta P_{\mathrm{t} 1} L}{2{\mathrm{ar}_{i}}\left[\frac{\nu_{1} \Pi_{\mathrm{ar}}^{2}}{F_{\mathrm{ar}}^{3}}+\frac{1_{\mathrm{v}} \Pi_{\mathrm{v}}^{2}}{F_{\mathrm{v}}^{3}}\right]} \tag{28}
\end{equation*}
$$

where $\ell_{\text {ari }}=\ell_{T T}-\left(0.5 \ell_{\text {in }}+x_{0 i}\right)$. A numerical method is then used to solve the system of equations (14)-(16), and we determine the corresponding $x_{0_{i}}$ value of $V_{e x c_{i}}$. The value of $\mathrm{Q}_{\max _{i}}$ at which $\mathrm{V}_{\mathrm{exc}}{ }_{i}$ is equal to the volume of the excess heat carrier, calculated in accordance with formula (20), represents the limit flow in the first portion of the heat-carrier circulation tract, i.e., $Q_{\max 1}$. The calculation of $Q_{\max 2}$ is accomplished in a manner similar to the calculation for the case in which $g=0$, with the only exception that the terms characterizing the gravitational boost in pressure in the calculation formulas are not equal to zero.

In Fig. 4 we can find the relationships $Q_{\max } T T=f(T)$ calculated on the basis of the above-described method for TT with freon-22 and ammonia as the heat-carrying agent. The TT exhibited the following parameters: $\ell_{T T}=1500 \mathrm{~mm}, \ell_{\mathrm{hs}}=\ell_{\mathrm{out}}=100 \mathrm{~mm}, \mathrm{~d}_{\text {in }}=11 \mathrm{~mm}, \mathrm{H}_{\mathrm{ar}}=$ $2 \mathrm{~mm}, \mathrm{~d}_{\mathrm{fla}}=0.7 \mathrm{~mm}, \mathrm{H}_{\mathrm{t}}=0.18 \mathrm{~mm}, \mathrm{~S}_{\mathrm{t}}=0.35 \mathrm{~mm}$, the opening angle of the threading grooves $2 \alpha=50^{\circ}, r=0.05 \mathrm{~mm}, \mathrm{~m}_{\mathrm{pr}}=31 \mathrm{~g}$ for Freon-22 and 15 g for ammonia. With $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{sec}^{2}$ the TT are horizontal.

Comparison of these results with those cited in [3] for the calculation of ammonia TT of approximately the same dimensions and for CS in the form of longitudinal grooves on the inside wall of the housing demonstrate that, for example, at a temperature of 300 K the limit flow of the arterial TT is greater by a factor of eight. This proves the high efficiency of the arterial CS. The difference in the limit flows for the cases in which $g=0$ and $g=$ $9.81 \mathrm{~m} / \mathrm{sec}^{2}$ may reach $50-100 \%$. This must necessarily be taken into consideration, evaluating the characteristics of the $T T$ when $g=0$, on the basis of the experimental results for the case in which $g=9.81 \mathrm{~m} / \mathrm{sec}^{2}$. Note should also be taken of the fact that the relationship between $Q_{\max }$ for two values of $g$ depends on the properties of the heat-carrying agent. For heat carriers with a relatively small $\sigma / \rho$ (Freon-22) we have $Q_{\max _{g}=0}>\mathrm{Q}_{\max }^{\mathrm{g}-9.81}$, while in the case of ammonia it is the opposite.

## NOTATION

Q, heat flow; $\nu$, kinematic viscosity coefficient, $\mathrm{m}^{2} / \mathrm{sec} ; \sigma$, coefficient of surface tension, $\mathrm{N} / \mathrm{m} ; \rho$, density, $\mathrm{kg} / \mathrm{m}^{3} ; \theta$, wetting angle, deg; L , heat of vaporization, $\mathrm{J} / \mathrm{kg} ; \mathrm{g}$, acceleration of free fall, $\mathrm{m} / \mathrm{sec}^{2} ; ~ \Pi$, perimeter, $\mathrm{m} ; \mathrm{S}$, thread interval, m ; \&, length, m ; d, diameter, $m ; r, r a d i u s, m ; \varepsilon$, porosity. Subscripts: max, maximum; in, inside; ar, arterial; liquid; $v$, vapor; exc, excess; fla, flange; hs, heat supply; out, outflow; t, thread; pr, priming; m, meniscus; j, joint; w, arterial wall; zo, zone; h, hydraulic; p, "puddle"; ori, orifice.

## LITERATURE CITED

1. I. M. Blinchevskii, B. F. Aptekar', and Ya. M. Baum, Prom. Teplotekh., 5, No. 6, 2629 (1983).
2. V. S. Tarasov, Inzh.-Fiz. Zh., 40, No. 2, 238-243 (1981).
3. S. Chi, Thermal Tubes. Theory and Practice [Russian translation], Moscow (1983).

[^0]:    Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 4, pp. 600-605, October, 1989. Original article submitted March 1, 1988.

